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Current Conservation, G_E vs. F_1, and Electromagnetic Interaction Currents for Non-relativistic Systems

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Abstract

It is shown how to construct non-relativistic interaction currents which satisfy current conservation. The construction permits the use of different electromagnetic form factors for the pion, nucleon, and contact contributions, and therefore also permits the use of either G_E or F_1. The numerical importance of this freedom for calculations of the electromagnetic form factors of ^{3}He is studied, and found to be significant in some cases.

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1. Introduction

In the calculation of electromagnetic observables, it is essential to use interaction currents which are consistent with gauge invariance. Feynman showed a number of years ago,[1] in the context of QED, that this required coupling the photon to all charged particles in all possible places in every diagram being calculated. In effective theories, where hadronic structure is treated phenomenologically through the introduction of strong form factors at meson-nucleon or meson-meson vertices, one does not have a theory for this hadronic structure, and Feynman's program cannot be carried out. This has lead to a popular misconception that either (i) a *full* theory of hadronic structure is necessary in order to have a gauge invariant current, or (ii) in the absence of such a theory, a *single* universal form factor must be used to describe the electromagnetic structure of the constituent hadrons.[2] This latter view is reinforced by the usual non-relativistic derivation of the interaction current from microscopic current conservation, where the nucleon form factors seem to play a special role through their connection with the one body charge density. Using only one form factor is, however, clearly in conflict with the physical idea that the total current is built up from one body (*i.e.* one nucleon) contributions, which should be described by nucleon form factors, meson-in-flight contributions, which should use meson form factors, and contact (or "pair") terms which might require still another form factor. Using the nucleon charge form factors to describe all of these processes is not only in conflict with our intuition, but has also lead to a debate over whether to use G_E or F_1 for these interaction currents.

Recently, in the context of relativistic meson theories, Gross and Riska[3] showed how to construct phenomenological interaction currents which (i) conserve current, even when phenomenological strong form factors are used at the meson-nucleon vertices, and (ii) place no restriction on the choice of phenomenological electromagnetic form factors used at each electromagnetic vertex, so that the appropriate (different) form factors may be used for nucleons, mesons, and contact terms (such as the $NN\pi\gamma$ term which arrises when $\gamma_5\gamma^\mu$

coupling is used for the pion). In this paper we apply their method to non-relativistic problems, and show how to construct non-relativistic interaction currents without constraints on the elementary electromagnetic form factors. The discussion gives insight into the role of the local non-relativistic current conservation relation, and shows where the original argument went astray. Our result allows us to settle the debate over the use of G_E vs F_1, and has significant numerical consequences for the present generation of three-body form factor calculations.

The method for taking the non-relativistic limit of a relativistic calculation is an integral part of the development, and in order to minimize confusion, the method will be described now. Our general philosophy is that the most appropriate frame in which to compare relativistic and non-relativistic calculations of electromagnetic form factors is the (generalized) Breit frame. If q is the four-momentum carried by the photon, this frame is defined by the conditions $q_0 = 0$ and $q^2 = -\mathbf{q}^2$, so that the photon matrix elements are independent of time and no energy is transferred to the target. The electromagnetic interaction in this frame is therefore static, consistent with the non-relativistic idea that the form factors describe how the bound state responds to static electric and magnetic field distributions, leading directly to the familiar interpretation of the form factors as charge and magnetic moment distributions.[4] Furthermore, in (non-covariant) time ordered perturbation theory, only the three-momentum of intermediate states is conserved, and therefore only the three-momentum transferred to the constituents is equal to the three-momentum transferred to the entire target. In covariant calculations, the full four-momentum of intermediate states is conserved, giving the result that the four momentum transferred to the (virtual, off-shell) constituents is always the same as the four-momentum transferred to the entire target. If a non-relativistic calculation is carried out in an arbitrary frame, this difference can lead to difficulty in deciding how to define the momentum transferred to the constituents,[5] and the Breit frame has the unique property that three-momentum conservation is sufficient to fix the actual photon momentum transfer correctly, permitting us to recover the (correct) relativistic result, eliminating ambiguities, and facilitating the comparison of relativistic and non-relativistic approaches. Finally, one should not be concerned that this procedure is frame dependent; because relativistic and nonrelativistic matrix elements satisfy different transformation laws a consistent comparison in all frames is impossible. With these remarks, we turn now to the main body of the paper.

2. The Single Nucleon Current

To set the stage, the role that the single nucleon current plays in the conservation of current is first reviewed briefly. The matrix element of the usual relativistic current operator of the nucleon is

$$\langle \mathbf{k}' \mid j_N^\mu(q) \mid \mathbf{k}\rangle = \left[\frac{m}{E(\mathbf{k}')}\frac{m}{E(\mathbf{k})}\right]^{1/2} \tag{1a}$$

$$\bar{u}(\mathbf{k}')e_p\left[F_1(q^2,\tau^3)\gamma^\mu + F_2(q^2,\tau^3)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(\mathbf{k}) \quad ,$$

$$F_1(0,\tau^3) = \frac{1}{2}(\tau^3+1) \quad , \tag{1b}$$

$$F_2(0,\tau^3) = \kappa(\tau^3) = \kappa_p\frac{1}{2}(\tau^3+1) + \kappa_n\frac{1}{2}(\tau^3-1) \quad , \tag{1c}$$

$$E(\mathbf{k}) = \left(m^2+\mathbf{k}^2\right)^{1/2} \quad , \tag{1d}$$

where k and k' are the four-momenta of the (virtual) initial and final nucleon, respectively, and $q = k' - k$ is the four-momentum which the virtual photon transfers to the nucleon. The form factor $F_2(q^2,\tau^3)$ is normalized to the anomalous magnetic moments of the proton κ_p and of the neutron κ_n. The nucleon spinors $u(\mathbf{k},\lambda) = u(\mathbf{k})$ describe physical on-shell nucleons with energy $E(\mathbf{k})$, momentum $\mathbf{k}$, and spin projection λ (which will normally be supressed). In applications to bound state problems, the nucleons are off-shell, so that their energies k_0 and k_0' are not equal to $E(\mathbf{k})$ and $E(\mathbf{k}')$ and the appearance of on-shell spinors is a consequence of the approximation in which a propagating virtual nucleon is replaced by its positive energy part (see the discussion following Eq. (13) below). This means that q_0 is not equal to $E(\mathbf{k}') - E(\mathbf{k})$, a reflection of the fact that, in a formalism where the (virtual) nuclear constituents are on-mass shell, the three-momentum is conserved, but the energy is *not* . [If we want to conserve energy, the constituents will be off-mass shell, as they are in Feynman diagrams. The connection between covariant Feynman diagrams and time ordered diagrams is at the heart of our discussion.]

Taking the non-relativistic limit of the above matrix element (in the Breit frame) gives

$$\frac{1}{e_p}\left\langle \mid j_N^0 \mid \right\rangle = F_1 - (F_1 + 2F_2)\left[\frac{\mathbf{q}^2}{8m^2} - \frac{i\sigma\cdot(\mathbf{q}\times\mathbf{k})}{4m^2}\right] \quad , \tag{2a}$$

$$\frac{1}{e_p}\left\langle \mid j_N^i \mid \right\rangle = F_1 \frac{(\mathbf{k}+\mathbf{k}')^i}{2m} - i\,(F_1 + F_2)\,\frac{(\mathbf{q}\times\sigma)^i}{2m} \quad , \tag{2b}$$

where terms of order m^{-3} have been discarded. Hence

$$\mathbf{q}^i \left\langle \mid \mathbf{j}_N^i \mid \right\rangle = e_p F_1(q^2, \tau^3)\frac{(\mathbf{k}'^2 - \mathbf{k}^2)}{2m} \tag{3a}$$

$$= \left[\kappa_0, \rho_{N_0}(q,\tau)\right] \tag{3b}$$

where κ is the kinetic energy operator and $\rho_N(q,\tau) = \left\langle \mid j_N^0 \mid \right\rangle$ is the nucleon charge operator, which in lowest order are

$$\kappa_0 = \frac{\mathbf{k}^2}{2m} \tag{4a}$$

$$\rho_{N_0}(q,\tau) = e_p F_1\left(q^2, \tau^3\right) \tag{4b}$$

Eq. (3b) looks like part of the non-relativistic continuity equation, and the occurrence of $F_1(q^2, \tau^3)$ in this equation has lead to much speculation about whether F_1 or G_E should be used in the interaction current. The fact that calculated results are sensitive to this difference has been taken to be evidence that relativistic effects (the difference between F_1 and G_E is of relativistic order) are important.

For comparison, consider the modified one-body current proposed in Ref. 3, which will be labeled by the subscript N' to distinguish it from (1),

$$\begin{aligned} \langle \mathbf{k}' \mid j^{\mu}_{N'}(q) \mid \mathbf{k} \rangle = & \left[\frac{m}{E(\mathbf{k}')} \frac{m}{E(\mathbf{k})} \right]^{1/2} \\ & \bar{u}(\mathbf{k}') e_p \left[F_1(q^2, \tau^3) \left(\gamma^{\mu} - \frac{\not{q}}{q^2} q^{\mu} \right) + F_1(0, \tau^3) \frac{\not{q}}{q^2} q^{\mu} \right. \\ & \left. + F_2(q^2, \tau^3) i \frac{\sigma^{\mu\nu} q_{\nu}}{2m} \right] u(\mathbf{k}) \end{aligned} \tag{5}$$

[Note that this current is *identical* to (1) *if the nucleons are on-shell*, and differs only when at least one nucleon is off shell.] This current satisfies the Ward-Takahashi (WT) identity[6] for a structureless nucleon. [In non-relativistic calculations using the Schrödinger equation, it is most consistent to use the identity for a structureless nucleon, but if nucleon structure is explicitly included in any calculation, the current (5) can be suitably modified, and the discussion generalized.] With our notation and normalization, the matrix elements of this identity are:

$$q_{\mu} \langle | \, j^{\mu}_{N'} \, | \rangle = \left[\frac{m}{E(\mathbf{k}')} \frac{m}{E(\mathbf{k})} \right]^{1/2} \bar{u}(\mathbf{k}') e_p F_1(0, \tau^3) \left[S^{-1}(k) - S^{-1}(k') \right] u(\mathbf{k}) \tag{6}$$

where

$$S^{-1}(k) = m - \not{k} \tag{7}$$

is the inverse of the free nucleon propagator. In the Breit frame, this equation becomes

$$\mathbf{q}^i \left\langle \mid \mathbf{j}^i_{N'} \mid \right\rangle = \left[\frac{m}{E(\mathbf{k}')}\frac{m}{E(\mathbf{k})}\right]^{1/2} \bar{u}(\mathbf{k}')e_p F_1\left(0,\tau^3\right)\left[\not{k} - \not{k}'\right]u(\mathbf{k}) \tag{8a}$$

$$= \left[E\left(\mathbf{k}'\right) - E\left(\mathbf{k}\right)\right]\rho_{NS}\left(q,\tau\right) = \left[\kappa, \rho_{NS}\left(q,\tau\right)\right] \tag{8b}$$

where the Dirac equation was used in going from (8a) to (8b), and the *exact* κ and ρ are

$$\kappa = E\left(\mathbf{k}\right) - m \tag{9a}$$

$$\rho_{NS}\left(q,\tau\right) = \left[\frac{m}{E(\mathbf{k}')}\frac{m}{E(\mathbf{k})}\right]^{1/2} e_p F_1\left(0,\tau^3\right)\bar{u}(\mathbf{k}')\gamma^0 u(\mathbf{k}) \tag{9b}$$

Note that $\rho_{NS}\left(q,\tau\right)$ is the exact charge density of a *point* particle, which is the reason for the subscript NS (for "no structure"). It is *not* equal to $\rho_{N'}(0,\tau) = \left\langle \mid j^0_{N'}(0,\tau) \mid \right\rangle$ (because, in the latter, the arguments of the spinors $\mathbf{k}$ and $\mathbf{k}'$ are equal). To lowest order in m^{-1}, Eq. (8) reduces to

$$\mathbf{q}^i \left\langle \mid \mathbf{j}^i_{N'} \mid \right\rangle = e_p F_1(0,\tau^3)\frac{\left(\mathbf{k}'^2 - \mathbf{k}^2\right)}{2m} \tag{10a}$$

$$= \left[\kappa_0, \rho_{N_0}\left(0,\tau\right)\right] \quad . \tag{10b}$$

Note that neither the exact relation (8) nor the lowest order relation (10) contain any reference to the internal structure of the nucleon. In fact, Eq. (10) no longer even suggests local current conservation, because in that relation the *total* charge appears on the RHS instead of the charge density, as in Eq. (3).

The choice of one body current operator therefore sets the stage for how we think about the problem of current conservation. The choice of Eq. (1) leads to Eq. (3), in which the divergence of the one body current is equal to the commutator of the kinetic energy operator with the local charge density of a nucleon with structure. Because the internal structure of the nucleon enters into this relation, we will refer to such statements as *microscopic* relations. On the other hand, the choice (5) for the one body current operator leads to a statement in which the divergence of the one body current is equal to the commutator of the kinetic energy operator with the local charge density of a *point* nucleon, given in Eq. (9*b*), and denoted by $\rho_{NS}(q, \tau)$. This density differs from the *total* charge of the nucleon [which is $\rho_N(0, \tau) = \rho_{N'}(0, \tau)$] only by terms of order m^{-2} compared to the leading term, and hence the difference between this charge density and the total charge will not be apparant in lowest order non-relativistic calculations. However, because (9*b*) is the exact charge *density* of a *point* charge, relations involving this charge density will be referred to as *macroscopic* relations.

In this language, we see that the choice of one body operator (5) has shifted the requirement of current conservation from a microscopic form (3) to a macroscopic form (8). One begins to see how it will be possible with this macroscopic form to avoid many of the issues raised by the microscopic form. In particular, $F_1(0, \tau^3) = G_E(0, \tau^3)$, so the issue of F_1 versus G_E does not even arise when using the macroscopic relations. Furthermore, since (8) is an exact relativistic relation, it may clearly be expanded in powers of m^{-2}, permitting relativistic effects to be treated systematically and easily.

Because the commutators (3) and (8) both involve the kinetic energy operators only, the proof of current conservation cannot be carried through. Other terms (interaction currents) are needed to insure current conservation in the general case. However, before discussing these additional currents, we return to the beginning, and show how the one body terms can be obtained from a fully relativistic treatment.

3. Non-relativistic Limit of One Body Currents

The five relativistic diagrams shown in Fig. 1 are a convenient starting point for our development. In Ref. 3 it was shown, in the context of the Bethe-Salpeter (BS) theory, that these five diagrams, taken together, conserve current when the two nucleons interact through the exchange of single bosons (the ladder approximation) even though the nucleons are off-shell and the initial and final state wave functions include interactions to all orders in the meson-nucleon coupling constant. The diagrams (a) and (b) lead to one body currents which contain the matrix elements (5). The interaction currents for a two body system are given completely by the three diagrams (c)–(e) shown in Fig. 1. These five diagrams will conserve current *provided* the form (5) is used for the one body current, *and* the other currents are constructed from the elementary currents given in Eqs. (33) and (34) below and in the Appendix. The general proof that these diagrams, defined in this way, lead to a conserved current, is given in Ref. 3, and will not be given here.

In this section we take the non-relativistic limit of the one body diagrams (a) and (b). The goal is to show in detail how the non-relativistic limit is to be taken, and how the off-shell nucleons are to be handled.

The two one body diagrams can be written

$$\left\langle\left\langle J^{[1]\mu}\right\rangle\right\rangle = \int\int \frac{d^4k_{12}' d^4k_{12}}{(2\pi)^8}$$

$$\bar{\Gamma}_f(k_{12}', K')S(k_1')S(k_2')J^{[1]\mu}\left({k_1}'{k_2}', k_1k_2\right) S(k_1)S(k_2)\Gamma_i(k_{12}, K) \qquad (11)$$

where

$$J^{[1]\mu}\left({k_1}'{k_2}', k_1k_2\right) = -i(2\pi)^4\left[\, j^{\mu}_{N'}(q)_1\; S_2^{-1}(k_2')\,\delta^4(k_2'-k_2)\right.$$
$$\left. +\, j^{\mu}_{N'}(q)_2\; S_1^{-1}(k_1')\,\delta^4(k_1'-k_1)\,\right] \quad (12)$$

and k_1 and k_2 are the momenta of the two incomming nucleons, k_1' and k_2' the momenta of the two outgoing nucleons (see Fig. 1c), S_i is the propagator (7) of the i^{th} nucleon, $J^{[1]\mu}$ is the relativistic one-nucleon interaction current, and $\Gamma_i(k_{12}, K)$ is the Bethe-Salpeter vertex function describing the coupling of the initial state (i) to two off-shell nucleons with relative momentum k_{12} and total momentum K, where, for both primed and unprimed momenta,

$K = k_1 + k_2$ and $k_{12} = \frac{1}{2}(k_1 - k_2)$. Note that the initial and final states need not be the same; the formalism is applicable to electrodisintegration processes as well as form factors (although numerical examples will be given only for form factors in this paper). Note that the $S^{-1}\ \delta^4$ factors in (12) eliminate the redundant propagator and $d^4k'_{12}$ integration in (11); they are included only so that the one body terms can be written in the same form as the two body terms (see below).

The details of how to take the nonrelativistic limit of (11), and express the result in terms of nonrelativistic wave functions, involves three steps:

(i) Removal of the relative energy variable $(k_{12})_0$ (the variable $(k'_{12})_0$ is eliminated by the δ function). The way in which this is done is critical when it is desired to calculate higher order relativistic effects, but these will not be discussed in this paper. All methods give the same lowest order, leading term.

(ii) Elimination of the negative energy components of the off-shell, propagating nucleon, and expression of the result in terms of positive energy nucleon states only.

(iii) Definition of the nonrelativistic wave function.

For the first step, it is sufficient to assume that, in the nonrelativistic limit, the dependence of the relativistic vertex functions, Γ_i and Γ_f, on the relative energies $(k_{12})_0$ and $(k'_{12})_0$ is very slowly varying compared to the rapid dependence of the propagators S_1 and S_2 on these variables. In practice, this means that we will evaluate all integrals ignoring any dependence on $(k_{12})_0$ and $(k'_{12})_0$ except that contained in S_1 and S_2. Hence the integrals over $(k_{12})_0$ (and $(k'_{12})_0$ for the interaction current terms discussed below) can be evaluated using the residue theorem. The second step is treated precisely by recalling that the relativistic nucleon propagator can be decomposed into positive and negative energy pieces

$$S(k) = \frac{m + \not{k}}{m^2 - k^2} = \frac{m}{E(\mathbf{k})} \sum_{\lambda} \left(\frac{u(\mathbf{k},\lambda)\bar{u}(\mathbf{k},\lambda)}{E(\mathbf{k}) - k_0} - \frac{v(-\mathbf{k},\lambda)\bar{v}(-\mathbf{k},\lambda)}{E(\mathbf{k}) + k_0} \right) \tag{13}$$

where the sum is over the spin states of the nucleon, and u and v are positive and negative energy spinors. The same physics which leads, in the nonrelativistic limit, to the assumption that the $(k_{12})_0$ and $(k'_{12})_0$ dependence can be localized in the nucleon propagator, also insures that the energy of the off-shell nucleons is always very close to the nucleon mass, m, so that the first term in the sum is usually of order m^2 larger than the second term. [Pion exchange with γ_5 coupling is an interesting exception to this general rule; in this case the coupling of v to u states is of order m times larger than the diagonal couplings, and negative energy states contribute to the lowest order relativistic corrections, and the analysis becomes much more complicated. In this discussion it will be assumed that the pion coupling is pure $\gamma_5\gamma^\mu$, and that the first term in (13) may be safely assumed to dominate the diagrams. In this case, the virtual nucleons may be described by pure positive energy states.] The factor $m/E(\mathbf{k})$ is divided symmetrically between the wave functions and the interaction (giving rise to the familiar factors of $[m/E(\mathbf{k})]^{1/2}$ which multiply each matrix element), and the energy denominator becomes part of the definition of the wave function. Then, introducing the nonrelativistic wave function Φ,

$$\Phi_i(\mathbf{k}_{12},\mathbf{K}) = -i\int \frac{d(k_{12})_0}{(2\pi)}\, S_1(k_1)\, S_2(k_2)\, \Gamma_i(k_{12},K) \tag{14a}$$

$$\simeq -i\int \frac{d(k_{12})_0}{(2\pi)} \left[\frac{m}{E(\mathbf{k}_1)}\frac{m}{E(\mathbf{k}_2)}\right]^{1/2} \frac{\bar{u}(\mathbf{k}_1)\bar{u}(\mathbf{k}_2)\Gamma_i(k_{12},K)}{(E(\mathbf{k}_1)-k_{10}-i\epsilon)(E(\mathbf{k}_2)-k_{20}-i\epsilon)} \tag{14b}$$

$$\simeq \left[\frac{m}{E(\mathbf{k}_1)}\frac{m}{E(\mathbf{k}_2)}\right]^{1/2} \frac{\bar{u}(\mathbf{k}_1)\bar{u}(\mathbf{k}_2)\Gamma_i(k_{12},K)}{(E(\mathbf{k}_1)+E(\mathbf{k}_2)-W)} \tag{14c}$$

where W is the total energy of the two nucleons, the contributions to the one body currents become

$$\left\langle\left\langle J^{[1]\mu}\right\rangle\right\rangle_{NR} = \int\int \frac{d^3k'_{12}d^3k_{12}}{(2\pi)^6}\,\bar{\Phi}_f(\mathbf{k}'_{12},\mathbf{K}')\left\langle J^{[1]\mu}_{N'}(k'_{12},k_{12})\right\rangle \Phi_i(\mathbf{k}_{12},\mathbf{K}) \tag{15a}$$

$$\left\langle J^{[1]\mu}_{N'}(k'_{12},k_{12})\right\rangle = (2\pi)^3\delta(\mathbf{k}'_2-\mathbf{k}_2)\left\langle \mathbf{k}'_1 \mid j^{\mu}_{N'} \mid \mathbf{k}_1\right\rangle + (2\pi)^3\delta(\mathbf{k}'_1-\mathbf{k}_1)\left\langle \mathbf{k}'_2 \mid j^{\mu}_{N'} \mid \mathbf{k}_2\right\rangle \ . \tag{15b}$$

Hence the result (8), when applied to the two body system, generalizes to:

$$\mathbf{q}^i\left\langle \mathbf{J}^{[1]i}_{N'}(k'_{12},k_{12})\right\rangle = \int \frac{d^3k''_{12}}{(2\pi)^3}\left[T(k'_{12},k''_{12})\rho^{[1]}_{NS}(k''_{12},k_{12}) - \rho^{[1]}_{NS}(k'_{12},k''_{12})T(k''_{12},k_{12})\right]$$

$$= \left[T,\rho^{[1]}_{NS}\right] \tag{16a}$$

where

$$\rho^{[1]}_{NS}(k'_{12},k_{12}) = (2\pi)^3\delta(\mathbf{k}'_2-\mathbf{k}_2)\rho_{NS}(q,\tau_1) + (2\pi)^3\delta(\mathbf{k}'_1-\mathbf{k}_1)\rho_{NS}(q,\tau_2) \tag{16b}$$

$$T(k'_{12},k_{12}) = (2\pi)^3\delta(\mathbf{k}'_{12}-\mathbf{k}_{12})(\kappa_1+\kappa_2) \tag{16c}$$

are the macroscopic one body charge and kinetic energy operators. The kinetic energy operator, T, has been expressed in the form it assumes when it is part of the kernel of the two body integral equation.

The wave equation satisfied by the amplitude (14) can be found from the non-relativistic reduction of the Bethe-Salpeter equation satisfied by the relativistic vertex function. The Bethe-Salpeter equation is

$$\Gamma_i(k'_{12},K) = i\int \frac{d^4k_{12}}{(2\pi)^4}\ V_I(k'_{12},k_{12};K)\ S(k_1)S(k_2)\Gamma_i(k_{12},K) \quad . \tag{17}$$

Multiplying both sides of (17) by $S(k_1')\ S(k_2')$, keeping only the positive energy parts of (13), and assuming that the dependence of V_I on $(k_{12})_0$ and $(k_{12}')_0$ is constant compared to the rapid variation of S_1S_2 (equivalent to the assumption that Γ is approximately constant), gives the equation for Φ

$$(E(\mathbf{k}_1') + E(\mathbf{k}_2') - W)\ \Phi(\mathbf{k}_{12}', \mathbf{K}) = -\int \frac{d^3k_{12}}{(2\pi)^3}\ V_{NR}(\mathbf{k}_1', \mathbf{k}_1; \mathbf{K})\ \Phi(\mathbf{k}_{12}, \mathbf{K}) \tag{18}$$

where the energy difference on the LHS comes from the exact evaluation of the integral over $[(E(\mathbf{k}_1') - k_{10}' - i\epsilon)(E(\mathbf{k}_2') - k_{20}' - i\epsilon)]^{-1}$, as in Eq. (14), and

$$V_{NR}(\mathbf{k}_1', \mathbf{k}_1; \mathbf{K}) = \left[\frac{m}{E(\mathbf{k}_1')}\,\frac{m}{E(\mathbf{K}-\mathbf{k}_1')}\,\frac{m}{E(\mathbf{k}_1)}\,\frac{m}{E(\mathbf{K}-\mathbf{k}_1)}\right]^{1/2}$$

$$\times\ \bar{u}(\mathbf{k}_1')\bar{u}(\mathbf{K}-\mathbf{k}_1')V_I(k_{12}', k_{12}; K)u(\mathbf{k}_1)u(\mathbf{K}-\mathbf{k}_1) \tag{19}$$

is the non-relativistic potential corresponding to the relativistic kernel. The dependence of this potential on the three-momenta has been given explicitly; it is specified by giving the initial and final momenta of particle 1 and the total momenta $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_1' + \mathbf{k}_2'$. Note that (in general) the non-relativistic potential will be local only to lowest order, even when the relativistic kernel is exactly local. For future reference, note that Eq. (18) can be rewritten

$$(W - 2m)\ \Phi(\mathbf{k}_{12}', \mathbf{K}) = \int \frac{d^3k_{12}}{(2\pi)^3}\ \left[T(k_{12}', k_{12}) + V_{NR}(\mathbf{k}_1', \mathbf{k}_1; \mathbf{K})\right]\ \Phi(\mathbf{k}_{12}, \mathbf{K}) \tag{20}$$

where the kinetic energy operator was defined in Eq. (16c).

4. Interaction Currents

We now turn to the treatment of the interaction currents, derivable from the diagrams (c)–(e) shown in Fig. 1. The sum of these three diagrams can be written in the same form as Eq. (11),

$$\left\langle\left\langle J^{[2]\mu}\right\rangle\right\rangle = -\int\int \frac{d^4k'_{12}d^4k_{12}}{(2\pi)^8}$$

$$\bar{\Gamma}_f(k'_{12},K')S(k'_1)S(k'_2)J^{[2]\mu}\left(k_1{}'k_2{}',k_1k_2\right)S(k_1)S(k_2)\Gamma_i(k_{12},K) \tag{21}$$

where $J^{[2]\mu}$ is the relativistic two-nucleon interaction current. It was shown in Ref. 3 that this interaction current could be written in the form

$$J^{[2]\mu}\left(k_1{}'k_2{}',k_1k_2\right) = ie_p\left(\tau_1\times\tau_2\right)^3\frac{(p'+p)^\mu}{p'^2-p^2}\left[V(p')-V(p)\right] \quad + \quad J_T^{[2]\mu} \tag{22}$$

where $p = k_2 - k'_2$, $p' = k'_1 - k_1$, $q = p' - p$, $J_T^{[2]\mu}$ is purely transverse $\left(q_\mu J_T^{[2]\mu} = 0\right)$, and V is related to the two-body BS kernel by

$$V_I = (\tau_1\cdot\tau_2)\,V \tag{23}$$

Here it is assumed that V depends only on the four-momentum transfer, which is the case for OBE interactions. In the Appendix, the form (22) is rederived, and the explicit form of $J_T^{[2]\mu}$ obtained, for the case of one pion exchange with a πNN vertex of the form $f_\pi(p^2)\,\gamma_5\gamma^\mu$, where f_π is an (arbitrary) πNN form factor and p^2 is the square of the four-momentum carried by the pion. For now these results will be assumed, the non-relativistic limit of (21) will be taken, and the implications for current conservation will be studied.

Following the steps outlined in the previous section, the integrations over $(k_{12})_0$ and $(k'_{12})_0$ can be quickly carried out, giving results similar to (15)

$$\left\langle\left\langle J^{[2]\mu}\right\rangle\right\rangle_{NR} = \int\int \frac{d^3k'_{12}d^3k_{12}}{(2\pi)^6}\bar{\Phi}_f(\mathbf{k}'_{12},\mathbf{K}')\left\langle J^{[2]\mu}(k'_{12},k_{12})\right\rangle \Phi_i(\mathbf{k}_{12},\mathbf{K}) \tag{24a}$$

$$\left\langle J^{[2]\mu}(k'_{12},k_{12})\right\rangle = \left[\frac{m}{E(\mathbf{k}'_1)}\frac{m}{E(\mathbf{k}'_2)}\frac{m}{E(\mathbf{k}_1)}\frac{m}{E(\mathbf{k}_2)}\right]^{1/2}\bar{u}(\mathbf{k}'_1)\bar{u}(\mathbf{k}'_2)J^{[2]\mu}u(\mathbf{k}_1)u(\mathbf{k}_2) \tag{24b}$$

Since we are interested in current conservation, we will first evaluate the divergence of this expression. Note that

$$q_\mu J^{[2]\mu} = ie_p\,(\tau_1\times\tau_2)^3\,[V(p')-V(p)] \tag{25a}$$

$$= \frac{1}{2}e_p\left(\left[(1+\tau_2^3)V_I(p')-V_I(p')(1+\tau_2^3)\right]+\left[(1+\tau_1^3)V_I(p)-V_I(p)(1+\tau_1^3)\right]\right). \tag{25b}$$

Using the completeness relation

$$\gamma^0 = \frac{m}{E(\mathbf{k})}\sum_\lambda\left[u(\mathbf{k},\lambda)\bar{u}(\mathbf{k},\lambda)+v(-\mathbf{k},\lambda)\bar{v}(-\mathbf{k},\lambda)\right] \tag{26}$$

permits the following reduction of (24b) in the Breit frame,

$$\mathbf{q}^i\left\langle\mathbf{J}^{[2]i}\right\rangle = -e_p\left[\frac{m}{E(\mathbf{k}'_1)}\frac{m}{E(\mathbf{k}'_2)}\frac{m}{E(\mathbf{k}_1)}\frac{m}{E(\mathbf{k}_2)}\right]^{1/2} \tag{27a}$$

$$\bar{u}(\mathbf{k}'_1)\bar{u}(\mathbf{k}'_2)\frac{1}{2}\left[(1+\tau_2^3)V_I(p')-V_I(p')(1+\tau_2^3)+(1+\tau_1^3)V_I(p)-V_I(p)(1+\tau_1^3)\right]u(\mathbf{k}_1)u(\mathbf{k}_2)$$

$$= -e_p \left[\frac{m}{E(\mathbf{k}_1')} \frac{m}{E(\mathbf{k}_2')} \frac{m}{E(\mathbf{k}_1)} \frac{m}{E(\mathbf{k}_2)} \right]^{1/2} \tag{27b}$$

$$\Bigg[\bar{u}(\mathbf{k}_2') \frac{1}{2}(1+\tau_2^3)\gamma^0 \left(\frac{m}{E(\mathbf{k}_2'-\mathbf{q})} \right) \sum_\lambda \Big(u(\mathbf{k}_2'-\mathbf{q},\lambda)\bar{u}(\mathbf{k}_2'-\mathbf{q},\lambda) \Big) \bar{u}(\mathbf{k}_1') V_I(p') u(\mathbf{k}_1) u(\mathbf{k}_2)$$

$$-\bar{u}(\mathbf{k}_1')\bar{u}(\mathbf{k}_2') V_I(p') u(\mathbf{k}_1) \left(\frac{m}{E(\mathbf{k}_2+\mathbf{q})} \right) \sum_\lambda \Big(u(\mathbf{k}_2+\mathbf{q},\lambda)\bar{u}(\mathbf{k}_2+\mathbf{q},\lambda) \Big) \gamma^0 \frac{1}{2}(1+\tau_2^3) u(\mathbf{k}_2)$$

$$+\bar{u}(\mathbf{k}_1') \frac{1}{2}(1+\tau_1^3)\gamma^0 \left(\frac{m}{E(\mathbf{k}_1'-\mathbf{q})} \right) \sum_\lambda \Big(u(\mathbf{k}_1'-\mathbf{q},\lambda)\bar{u}(\mathbf{k}_1'-\mathbf{q},\lambda) \Big) \bar{u}(\mathbf{k}_2') V_I(p) u(\mathbf{k}_1) u(\mathbf{k}_2)$$

$$-\bar{u}(\mathbf{k}_1')\bar{u}(\mathbf{k}_2') V_I(p) u(\mathbf{k}_2) \left(\frac{m}{E(\mathbf{k}_1+\mathbf{q})} \right) \sum_\lambda \Big(u(\mathbf{k}_1+\mathbf{q},\lambda)\bar{u}(\mathbf{k}_1+\mathbf{q},\lambda) \Big) \gamma^0 \frac{1}{2}(1+\tau_1^3) u(\mathbf{k}_1) \Bigg]$$

$$= -\rho_{NS}(q,\tau_2) V_{NR}(\mathbf{k}_1',\mathbf{k}_1;-\frac{1}{2}\mathbf{q}) + V_{NR}(\mathbf{k}_1',\mathbf{k}_1;\frac{1}{2}\mathbf{q})\rho_{NS}(q,\tau_2) \tag{27c}$$

$$- \rho_{NS}(q,\tau_1) V_{NR}(\mathbf{k}_1'-\mathbf{q},\mathbf{k}_1;-\frac{1}{2}\mathbf{q}) + V_{NR}(\mathbf{k}_1',\mathbf{k}_1+\mathbf{q};\frac{1}{2}\mathbf{q})\rho_{NS}(q,\tau_1)$$

$$= \int \frac{d^3k''}{(2\pi)^3} \left[V_{NR}(\mathbf{k}_1',\mathbf{k}_1'';\frac{1}{2}\mathbf{q}) \rho_{NS}^{[1]}(k_{12}'',k_{12}) - \rho_{NS}^{[1]}(k_{12}',k_{12}'') V_{NR}(\mathbf{k}_1'',\mathbf{k}_1;-\frac{1}{2}\mathbf{q}) \right] \tag{27d}$$

$$= \left[V_{NR}, \rho_{NS}^{[1]} \right] \tag{27e}$$

where, in going from the first to the second step, we used $\gamma^0\gamma^0 = 1$ with one γ^0 replaced by the identity (26) (with the v spinor terms discarded) and, in the last step, used the one-body charge operator from Eq. (16b). Combining this result with the result for the one-body current, (16a), gives the nonrelativistic demonstration of current conservation we have been seeking:

$$\mathbf{q}^i \left\langle \left\langle \left(\mathbf{J}^{[1]i} + \mathbf{J}^{[2]i} \right) \right\rangle \right\rangle_{NR} = \left\langle \left[(T + V_{NR}), \rho_{NS}^{[1]} \right] \right\rangle_{NR}$$

$$= \left\langle \left[H, \rho_{NS}^{[1]} \right] \right\rangle_{NR} \quad = \quad 0 \quad , \tag{28}$$

where the outer $\langle\ \rangle_{NR}$ brackets represent the matrix element of the wave functions, (14), and to get zero in the final step use the wave equation (20) and the fact that in the Breit frame the initial and final energies of the two body system are equal. Note that the interaction currents extend the macroscopic conservation relation without placing restrictions on the hadron electromagnetic form factors used in the calculation.

We emphasize that there will be relativistic corrections to (28) comming from negative energy states, which have been consistently neglected in this development. A proper discussion of these effects really requires a relativistic treatment, and would follow the lines outlined in Ref. 3 and Ref. 7. Before leaving this topic, we briefly discuss one important class of such contributions. Consider the diagrams shown in Fig. 2 *a–d*. Since the relativistic wave equation states that the bound state vertex function is equal to the convolution of the one-boson exchange (OBE) interaction with the bound state vertex function, as illustrated in Fig. 2*e*, these diagrams are equal to twice the one body diagrams shown in Fig. 1*a* and *b*, if the *exact* bound state vertex function is used, and therefore make no additional contribution to the two body current. However, in a non-relativistic approximation where the v spinor contributions to the relativistic wave equation are neglected, the contributions to the nucleon propagators comming from the v spinor sums in Eq. (13) are neither contained in the vertex functions nor in the one-body current defined in Eq. (15b), and are therefore new, hitherto ignored terms which must be added explicitly to the two-body current. In general, the difference between the full field theoretic current derived from the Born terms of Fig.2 and the nonrelativistic one-body contributions is a new irreducible two-nucleon interaction current, which can be denoted $J^{[2]\mu}_{\mathrm{Born}}(q)$. The total interaction current is then

$$J^{[2]\mu}(q) = \left[J^{\mu}_{\mathrm{Born}}(q) - J^{[1]\mu}(q)\right] + J^{[2]\mu}_{\pi}(q) + J^{[2]\mu}_{C}(q) \quad , \tag{29a}$$

$$= \qquad J^{[2]\mu}_{\mathrm{Born}}(q) + J^{[2]\mu}_{\pi}(q) + J^{[2]\mu}_{C}(q) \quad . \tag{29b}$$

where $J_\pi^{[2]\mu}$ and $J_C^{[2]\mu}$ are the full two body contributions from the " pion-in-flight " diagram, Fig. 1*c*, and the contact diagrams, Fig. 1*d*–*e*. For $\gamma_5\gamma^\mu$ coupling the new term, $J^\mu_{\rm Born}$, does not contribute to leading order in m^{-1}, but it is important (comparable in size to J_C) for γ_5 coupling. We will not discuss these issues further.

Armed with the knowledge that the total current is conserved, both in the relativistic and nonrelativistic cases, we are now ready to find the nonrelativistic limit of the interaction current for use in nonrelativistic calculations. The longitudinal part, $J_L^{[2]\mu} = J^{[2]\mu} - J_T^{[2]\mu}$, can be reduced using the same steps used in Eq. (27). To lowest order in m^{-1}, it is purely space-like, and we obtain

$$\left\langle \mathbf{J}_L^{[2]i} \right\rangle = ie_p\,(\tau_1 \times \tau_2)^3\,\frac{(\mathbf{p}+\mathbf{p}')^i}{\mathbf{p}^2-\mathbf{p}'^2}\,(\mathbf{V}_{NR}(\mathbf{p}') - \mathbf{V}_{NR}(\mathbf{p})) \tag{30}$$

where use was made of the fact that the nonrelativistic limit of a one pion exchange potential is local, and the notation $\mathbf{V}_{NR}(\mathbf{p}) = V_{NR}(\mathbf{k}_2', \mathbf{k}_2)$ will be used. Specifically, the nonrelativistic one pion exchange potential can be written in the following form:

$$\mathbf{V}_{NR}(\mathbf{p}) = 3\ \sigma_1\!\cdot\!\mathbf{p}\ \sigma_2\!\cdot\!\mathbf{p}\ v(\mathbf{p}) \tag{31a}$$

$$= -\sigma_1\!\cdot\!\mathbf{p}\ \sigma_2\!\cdot\!\mathbf{p}\ \left(\frac{g}{2M}\right)^2 \Delta(-\mathbf{p}^2) \tag{31b}$$

where Δ is the full relativistic pion propagator, which includes all effects from hadronic form factors at the πNN vertices. Specifically, if f_π is the pion form factor and p^2 is the square of the virtual pion four-momentum, then

$$\Delta(p^2) = \frac{[f_\pi(p^2)]^2}{p^2 - m_\pi^2} \quad . \tag{32}$$

In the nonrelativistic limit, $p^2 = -\mathbf{p}^2$, giving the familiar Yukawa potential, with form factors. However, since this derivation works for *any* form factor f_π, Δ may be replaced in applications by a phenomenological potential.

The specific form of the transverse part of the pion current has been worked out in the Appendix and its nonrelativistic limit is also obtained there. Its construction requires introduction of a generalized pion current, to be used in the calculation of the diagram in Fig. 1c, and a contact current to describe the interaction at the $\gamma\pi NN$ vertex, needed to calculate the diagrams in Figs. 1d and e. The pion current is

$$j_{\pi}^{ji\nu}(q) = f_{\pi}(p'^2) j_{R\pi}^{ji\nu}(q) f_{\pi}(p^2) \tag{33a}$$

$$j_{R\pi}^{ji\nu}(q) = -ie_p\varepsilon_{ji3}\frac{\Delta^{-1}(p'^2)-\Delta^{-1}(p^2)}{p'^2-p^2}\Big[F_{\pi}(q^2)(p'+p)^{\nu}$$

$$-[F_{\pi}(q^2)-F_{\pi}(0)]\frac{(p'+p)\cdot q}{q^2}q^{\nu}\Big] \quad , \tag{33b}$$

$$F_{\pi}(0) = 1 \quad , \tag{33c}$$

and the contact current is

$$j_{C}^{j\nu}(q) = f_{\pi}(p'^2) j_{RC}^{j\nu} \tag{34a}$$

$$j_{RC}^{j\nu} = e_p\frac{g}{2m}\varepsilon_{ji3}\tau_i\Big[F_C(q^2)\gamma_5\gamma^{\nu} - [F_C(q^2)-F_C(0)]\gamma_5\frac{\not q}{q^2}q^{\nu}\Big] \quad , \tag{34b}$$

$$F_C(0) = 1 \quad . \tag{34c}$$

In (33), i and j are the isospin lables of the incoming and outgoing pion, respectively, $F_{\pi}(q^2)$ is the form factor of the pion, and $j_{R\pi}^{ji\nu}(q)$ is the reduced current which must satisfy the WT identity. The current includes the full pion propagator, $\Delta(p^2)$, and the πNN form factor, f_{π}, discussed above. In (34), the index j is the isospin of the pion, which is outgoing, $\boldsymbol{\tau}$ is the isospin operator of the nucleon, and a form factor for the contact process $F_C(q^2)$ has been introduced. All form factors can be chosen independently from each other as in Ref. 3. Note that the reduced pion current (33b) satisfies the WT identity for a pion with the modified propagator Δ:

$$q_\nu j^{ji\nu}_{R\pi} = -ie_p \varepsilon_{ji3}\left[\Delta^{-1}(p'^2) - \Delta^{-1}(p^2)\right] \tag{35}$$

while the divergence of the reduced contact current is not changed compared to the case without a form factor.

Using the operators (33) and (34), it is shown in the Appendix that the transverse part of the two-body current becomes:

$$\left\langle \mathbf{J}_T^{[2]i}\right\rangle = 3ie_p\left(\tau_1\times\tau_2\right)^3$$

$$\times\left[F_C(q^2)\left(\sigma_1\cdot\mathbf{p}'\left[\sigma_2^i - \sigma_2\cdot\mathbf{q}\frac{\mathbf{q}^i}{\mathbf{q}^2}\right]v(\mathbf{p}') + \sigma_2\cdot\mathbf{p}\left[\sigma_1^i - \sigma_1\cdot\mathbf{q}\frac{\mathbf{q}^i}{\mathbf{q}^2}\right]v(\mathbf{p})\right)\right.$$

$$\left.+\left[\frac{(\mathbf{p}'+\mathbf{p})^i}{\mathbf{p}'^2-\mathbf{p}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2}\right]\left(F_\pi(q^2)\sigma_1\cdot\mathbf{p}'\sigma_2\cdot\mathbf{p}[v(\mathbf{p}')-v(\mathbf{p})] + \frac{1}{3}\left[\mathbf{V}_{NR}(\mathbf{p}')-\mathbf{V}_{NR}(\mathbf{p})\right]\right)\right] \tag{36}$$

In using this current in electron scattering calculations, the terms proportional to $\mathbf{q}^i$ can be dropped, because they give zero when contracted with the conserved electron current. In this case the result simplifies, and the longitudinal part, (30), is cancelled by an identical term in the transverse part, giving two contributions similar to the standard pion contact and pion current terms familiar from previous non-relativistic calculations

$$\left\langle \mathbf{J}^{[2]i}\right\rangle = 3ie_p\left(\tau_1\times\tau_2\right)^3\left[F_C(q^2)\Big(\sigma_1\cdot\mathbf{p}'\,\sigma_2^i v(\mathbf{p}') + \sigma_2\cdot\mathbf{p}\sigma_1^i v(\mathbf{p})\Big)\right.$$

$$\left.+F_\pi(q^2)\sigma_1\cdot\mathbf{p}'\sigma_2\cdot\mathbf{p}\Big(v(\mathbf{p}')-v(\mathbf{p})\Big)\frac{(\mathbf{p}'+\mathbf{p})^i}{\mathbf{p}'^2-\mathbf{p}^2}\right] \quad . \tag{37}$$

To summarize: the complete current $J^{\mu}(q) = J^{[1]\mu} + J^{[2]\mu}$ of the interacting two-nucleon system of Fig. 1 is composed of two parts; the nucleon current, defined relativistically in Eq. (5), and nonrelativistically in Eq. (15b), and the two-body interaction current given relativistically in Eq. (22), and nonrelativistically in Eq. (37) and explicitly constructed from its component parts, j_{π} (33) and j_C (34), in the Appendix. It is conserved, i.e.,

$$q_{\mu}\,\langle\langle J^{\mu}\rangle\rangle_{NR} = \int\int \frac{d^3k'_{12}d^3k_{12}}{(2\pi)^6}\bar{\Phi}_f(\mathbf{k}'_{12},\mathbf{K}')\,\langle q_{\mu}J^{\mu}(k'_{12},k_{12})\rangle\,\Phi_i(\mathbf{k}_{12},\mathbf{K})$$

$$= 0 \quad , \tag{38}$$

even though the electromagnetic form factors in the various processes of Fig. 1 as well as the hadronic πNN form factor are chosen independently. This result is due to the WT identities, Eqs.(6) and (35), and the way in which the elementary currents (5), (33) and (34) are defined.

This result is in conflict with Ref. 8. In this reference a pion current of the same form as (37) is used (their Eq. (5) with p and p' replaced by $-k_2$ and k_1), but both terms are multiplied by a common form factor, G_E, which they state is required by current conservation. While such a choice is permitted by our result, it is certainly *not* required. In particular, since the pion form factor is known to fall more slowly than G_E, the choice of G_E would *not* seem to be a choice most consistent with the meson exchange model. Furthermore, the contact current should more properly go like F_C or F_1, which are also much larger than G_E. Thus the choice made in Ref. 8 can be expected to underestimate the effect of interaction currents (see the discussion in Section 6 below).

5. Microscopic Current Conservation

We saw in Secs. 2–4 how the non-relativistic reduction of the relativistic relations leads naturally to the appearance of current conservation in the macroscopic form, Eq. (28). In non-relativistic physics, current conservation is usually expressed as a microscopic relation:

$$\mathbf{q}^i \left\langle \left\langle \left(\mathbf{J}^{[1]i} + \mathbf{J}^{[2]i} \right) \right\rangle \right\rangle_{NR} = \left\langle \left[H, \rho_N^{[1]}(q) \right] \right\rangle_{NR} \quad = \quad 0 \tag{39}$$

where $\rho_N^{[1]}(q)$ is the local charge operator obtained by substituting the charge density obtained from Eq. ($1a$) into ($16b$). In this section we show that the conservation relations may also be re-expressed in such a microscopic form, and discuss the significance of this result.

To show this, first *add and subtract* the following interaction term:

$$J_{L'}^{[2]\mu} = ie_p \left[\tau_1 \times \tau_2\right]^3 \frac{q^\mu}{q^2} \left[F_1(q^2) - 1\right] \left[V(p') - V(p)\right] \quad . \tag{40}$$

Note that this current is acceptable because it is regular at $q^2 = 0$. Next, replace the one body current $J_{N'}^{[1]\mu}(q)$ of Eq. (15b) with the one body current $J_N^{[1]\mu}(q)$, obtained from (15b) by substituting $j_N^\mu(q)$ of Eq. (1) for $j_{N'}^\mu(q)$ of Eq. (5), and regard the difference, $J_{N'}^{[1]\mu}(q) - J_N^{[1]\mu}(q)$, as a contribution to the interaction current. With these changes, the interaction current can now be written

$$J^{[2]\mu} = ie_p \left[\tau_1 \times \tau_2\right]^3 F_1(q^2) \frac{(p' - p)^\mu}{p'^2 - p^2} \left[V(p') - V(p)\right] - J_{L'}^{[2]\mu} + \left[J_{N'}^{[1]\mu}(q) - J_N^{[1]\mu}(q)\right] + J_{T'}^{[2]\mu} \tag{41}$$

where the new transverse part of the current is

$$J_{T'}^{[2]\mu} = -ie_p\,[\tau_1 \times \tau_2]^3 \left(\frac{P^\mu}{P\cdot q} - \frac{q^\mu}{q^2}\right)\left[F_1(q^2) - 1\right]\left[V(p') - V(p)\right] \;+\; J_T^{[2]\mu}\;. \qquad (42)$$

Now, the new longitudinal pieces in Eq. (41) depend on q^μ only, and hence will be zero when contracted with the electron current (or, in the case of photoproduction processes, with the polarization vector of the real photon). If these terms are discarded, the sum of the new one body and two body interaction currents satisfies the microscopic relation (39)!

In summary, we are able to convert the statement of current conservation from a macroscopic form to a microscopic form by adding and subtracting the term (40), and then dropping some terms proportional to q^μ. These latter terms make no contribution to any physical interaction, and do not change the final result for the interaction current, but they are important in showing us why current conservation alone will place no constraints on the form factors used in each term in the current. If the structure of these terms is artifically constrained (by assuming they must be zero, for example) we may be lead to conclude that current conservation requires certain restrictions which, in fact, are not required. It is the freedom to introduce such terms, and their natural appearance in the construction of currents which satisfy the WT identities, which relaxes the commonly assumed restrictions on the choice of form factors.

6. Numerical Sensitivity

In non-relativistic calculations it is often customary to replace the Dirac and Pauli form factors of the nucleon by the Sachs form factors, i.e.,

$$G_E(q^2,\tau^3) = F_1(q^2,\tau^3) - \frac{q^2}{4m^2}F_2(q^2,\tau^3) \quad , \qquad (43a)$$

$$G_M(q^2,\tau^3) = F_1(q^2,\tau^3) + F_2(q^2,\tau^3) \quad , \qquad (43b)$$

which are interpreted as experimental quantities of order $(1/m)^0$. These are the appropriate form factors to use in the one body (nucleon) currents. However, as we have emphasized, the experimental pion form factor[9,10] may be used in the pion exchange processes, and the axial form factor [11]

$$F_C(q^2) = \frac{1}{(1 - q^2/m_A^2)^2} \tag{44}$$

may be used for the contact process, *without violating gauge invariance*. To study the numerical importance of these replacements, the pion form factor of Gari and Krümpelmann[11] and form factor (44) with $m_A = 1.09$ GeV will be used.

Figures 3 and 4 give some indication of how the theoretical conclusions of this paper can be expected to influence practical numerical results. In Fig. 3, the form factors F_1^V, G_E^V, F_π, and F_C are compared. For the nucleon form factors, the parametrization of Blatnik and Zovko[12] is used. Note that only G_E^V is significantly different from the others over the momentum range of relevance to the three body form factors. Hence, the use of F_π and F_C in place of F_1 will not introduce large changes, but the replacement of G_E^V by F_π or F_C could introduce large effects.

The analysis of the two-body exchange currents of Sec.4 has bearing on the trinucleon magnetic properties only. Predictions with the new choice of operators in Sec.4 are compared in Fig. 4 with the results of Ref. 13. In this reference, the electromagnetic form factors of the individual nuclear constituents, i.e., the ones needed for the single-baryon charge and for the two-baryon spatial meson-exchange currents required by current conservation, were equated to the Sachs form factor G_E^V (given by the short dashed line in Figure 4). Substituting the realistic pion form factor for G_E^V in the pion exchange term (dotted line) has very little effect, even though the pion form factor differs significantly from G_E^V in this momentum transfer range (see Fig. 3). This is a reflection of the fact that the pion exchange term is not very important. However, substituting F_C for G_E^V in the pion contact interaction (the long dashed line) has a significant effect, a reflection of the previously known fact that the contact term makes important contributions to the three body form factors. Finally, the solid curve, which shows the result when F_1^V is used for all of the pion and rho exchange currents, shows a further sensitivity to the choice of rho form factors, which we have not investigated in this paper, but which could be studied using the methods we have developed.

6. Conclusions

In this paper we show how to construct two-body interaction currents for non-relativistic systems. The interaction currents insure that all two body matrix elements of the total current are conserved, even when different electromagnetic form factors are used for the nucleon, pion, and contact terms. While this paper presents detailed formulae only for pion exchange currents derivable from the pseudo-vector πNN coupling, results for other exchange currents can be derived in a similar fashion.

The currents are obtained by taking the non-relativistic limit of the relativistic currents and matrix elements previously derived in Ref. 3. The non-relativistic limit is taken in the generalized Breit frame, where the energy component of the relativistic four-momentum transfer, q, is zero. This frame is preferred because only in this frame is the four-momentum transfer of relativistic physics equal to the three-momentum transfer of non-relativistic physics, making it possible to make a detailed correspondence between the two. The derivation leads to several new observations about the relationship between relativistic and non-relativistic theories:

(i) The Ward-Takahashi identity for the nucleon, which plays a central role in the derivation of the relativistic interaction current, reduces to the statement that the divergence of the one body current operator is equal, in the Breit frame, to the commutator of the kinetic energy operator with the *macroscopic* charge operator [Eqs. (8) and (16)].

(ii) The divergence of the interaction current reduces, in the Breit frame, to an equation relating the divergence of the interaction current to the commutator of the potential energy operator with the *macroscopic* charge operator [Eq. (27)].

(iii) Combining items (i) and (ii) above shows that the divergence of the total current [which is the sum of the one body currents and the interaction (two body) currents] is equal to the commutator of the hamiltonian with the *macroscopic* charge operator. Since the matrix element of this commutator is zero, the total current is conserved.

(iv) The appearance of the *macroscopic* charge operator in the commutation relations referred to in items (i) - (iii) above [which we have called "macroscopic" relations], instead of the usual charge density [called "microscopic" relations], appears to be a natural consequence of the relativistic structure of the currents, and shows why the choice of nucleon structure plays no role in the statement of current conservation.

A major conclusion of this work, which follows particularly from the last item above, is that the form factor used for the contact term may be either G_E, F_1, or, more generally, F_C, and that this choice is neither restricted by the requirements of current conservation, nor by whether one wishes to use G_E or F_1 for the single nucleon form factor. For example, if the currents are properly constructed, one may use F_1 for the contact form factor and G_E for the single nucleon charge operator (which may be preferred by the data). It has been known for some time that the choice of this form factor can be numerically significant, and this fact is demonstrated again in Fig. 4, which shows the sensitivity of the three-nucleon magnetic form factors to this choice. These observations show that meson theories have less predictive power than commonly assumed, and suggest that the contact form factor, at least within the context of non-relativistic calculations, can be chosen to fit data. We conclude that the choice made in Ref. 8 to use G_E for the one body form factors in the interaction current is not a unique choice forced by the requirements of current conservation, but, in view of the success of the fits, was a good choice for that calculation.

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Appendix: Form of the Pion Interaction Current

In this Appendix we show that the interaction current constructed from Figs. 1c–e can be written in the form given in Eq. (22), and obtain the exact form for the transverse term, $J_T^{[2]\mu}$. The construction uses the elementary pion and contact currents given in Eqs. (33) and (34), and is thus valid for $\gamma_5\gamma^\mu$ pion coupling with an arbitrary form factor $f_\pi(p^2)$, where p^2 is the four momentum squared of the virtual pion. After determining the exact relativistic form of $J_T^{[2]\mu}$, we take the non-relativistic limit and obtain Eq. (36).

Using the Feynman rules, the current in Fig. 1c describing the interaction with the "pion-in-flight" is

$$J_\pi^{[2]\mu} = -i\varepsilon_{ji3}\tau_1^i\tau_2^j\left(\frac{g}{2m}\right)^2[\not p'\gamma_5]_1\,[\not p\,\gamma_5]_2\,\Delta(p^2)\Delta(p'^2)$$

$$\left[F_\pi(q^2)\frac{\Delta^{-1}(p'^2)-\Delta^{-1}(p^2)}{p'^2-p^2}\left(P^\mu-\frac{P\cdot q\,q^\mu}{q^2}\right)+\left(\Delta^{-1}(p'^2)-\Delta^{-1}(p^2)\right)\frac{q^\mu}{q^2}\right] \quad (A.1a)$$

$$= -i\,[\tau_1\times\tau_2]^3\left(\frac{g}{2m}\right)^2[\not p'\gamma_5]_1\,[\not p\,\gamma_5]_2\left[-P^\mu\;\left(\frac{\Delta(p'^2)-\Delta(p^2)}{p'^2-p^2}\right)\right.$$

$$\left.-\,(F_\pi(q^2)-1)\left(P^\mu-\frac{P\cdot q\,q^\mu}{q^2}\right)\frac{\Delta(p'^2)-\Delta(p^2)}{p'^2-p^2}\right] \qquad (A.1b)$$

where $P = p' + p$ and all other terms were defined in Sec. 4. Similarily, the two diagrams 1d and 1e are:

$$J_C^{[2]\mu} = i\,[\tau_1 \times \tau_2]^3 \left(\frac{g}{2m}\right)^2 \Bigg\{ [\not{p}\,\gamma_5]_2\, \Delta(p^2) \left([\gamma^\mu\, \gamma_5] + (F_C(q^2)-1) \left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) \gamma_5 \right)_1$$

$$+ [\not{p}'\, \gamma_5]_1\, \Delta(p'^2) \left([\gamma^\mu\, \gamma_5] + (F_C(q^2)-1) \left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) \gamma_5 \right)_2 \Bigg\} \qquad (A.2a)$$

$$= i\,[\tau_1 \times \tau_2]^3 \left(\frac{g}{2m}\right)^2 \Bigg\{ \Big([\not{p}\,\gamma_5]_2\, [\not{q}\,\gamma_5]_1\, \Delta(p^2) + [\not{p}'\,\gamma_5]_1\, [\not{q}\,\gamma_5]_2\, \Delta(p'^2) \Big) \frac{P^\mu}{P\cdot q}$$

$$+ [\not{p}\,\gamma_5]_2\, \Delta(p^2) \left([\gamma^\mu\, \gamma_5] - [\not{q}\gamma_5]\, \frac{P^\mu}{P\cdot q} + (F_C(q^2)-1) \left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) \gamma_5 \right)_1$$

$$+ [\not{p}'\, \gamma_5]_1\, \Delta(p'^2) \left([\gamma^\mu\, \gamma_5] - [\not{q}\gamma_5]\, \frac{P^\mu}{P\cdot q} + (F_C(q^2)-1) \left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) \gamma_5 \right)_2 \Bigg\} \qquad (A.2b)$$

Adding $(A.1)$ and $(A.2)$ together gives a result of the following form

$$J^{[2]\mu} = i\,[\tau_1 \times \tau_2]^3 \left(\frac{g}{2m}\right)^2 \frac{P^\mu}{P\cdot q} \Bigg\{ [\not{p}'\gamma_5]_1\, [\not{p}\gamma_5]_2 \left(\Delta(p'^2) - \Delta(p^2) \right)$$

$$+ [\not{p}\gamma_5]_2\, [\not{q}\,\gamma_5]_1\, \Delta(p^2) + [\not{p}'\,\gamma_5]_1\, [\not{q}\,\gamma_5]_2\, \Delta(p'^2) \Bigg\}$$

$$+ \{transverse \quad terms\} \qquad (A.3)$$

where the transverse terms are the last terms in the expressions $(A.1b)$ and $(A.2b)$. Substituting $q = p' - p$ into the longitudinal terms in $(A.3)$, and defining V of Eq. (23) by

$$V(p) = \left(\frac{g}{2m}\right)^2 [\not{p}\gamma_5]_1\, [\not{p}\gamma_5]_2\, \Delta(p^2) \qquad (A.4)$$

gives Eq. (22), with

$$J_T^{[2]\mu} = i\,[\tau_1 \times \tau_2]^3 \left\{ \left(P^\mu - \frac{P\cdot q\, q^\mu}{q^2} \right) R \right.$$

$$\left. + \left(\frac{g}{2m}\right)^2 F_C(q^2) \left[[\not{p}\gamma_5]_2\, \Delta(p^2) \left([\gamma^\mu\gamma_5]_1 - [\not{q}\gamma_5]_1 \frac{q^\mu}{q^2} \right) + [\not{p}'\gamma_5]_1\, \Delta(p'^2) \left([\gamma^\mu\gamma_5]_2 - [\not{q}\gamma_5]_2 \frac{q^\mu}{q^2} \right) \right] \right\} \qquad (A.5)$$

and

$$R = -\left(\frac{V(p') - V(p)}{p'^2 - p^2} \right) + F_\pi(q^2) \left(\frac{g}{2m}\right)^2 [\not{p}'\gamma_5]_1\, [\not{p}\gamma_5]_2\, \frac{\Delta(p^2) - \Delta(p'^2)}{p'^2 - p^2} \qquad (A.6)$$

The next task is to evaluate the nonrelativistic limit of the interaction current in the Breit frame. To this end we use the following matrix elements:

$$\begin{aligned}
\bar{u}(\mathbf{k}_1')\,[\not{p}'\gamma_5]_1\, u(\mathbf{k}_1) &\approx -\boldsymbol{\sigma}_1\cdot\mathbf{p}' \\
\bar{u}(\mathbf{k}_2')\,[\not{p}\,\gamma_5]_2\, u(\mathbf{k}_2) &\approx -\boldsymbol{\sigma}_2\cdot\mathbf{p} \\
\bar{u}(\mathbf{k}_1')\,\left[\gamma^i\gamma_5\right]_1 u(\mathbf{k}_1) &\approx \boldsymbol{\sigma}_1^{\mathbf{i}} \\
\bar{u}(\mathbf{k}_2')\,\left[\gamma^i\gamma_5\right]_2 u(\mathbf{k}_2) &\approx \boldsymbol{\sigma}_2^{\mathbf{i}}
\end{aligned} \qquad (A.7)$$

and note that, in the Breit frame, $q^2 = -\mathbf{q}^2$, and $q^\mu \to \mathbf{q}^i$ (where $\mu = i = 3$). With these identities, the non-relativistic limit of $(A.5)$ becomes Eq. (36).

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FIGURE CAPTIONS

Fig. 1 The five diagrams which give, for the BS theory in ladder approximation, a gauge invariant calculation of the electromagnetic interactions of two nucleons. (a) and (b) are one body terms, (c) the meson exchange process, and (d) and (e) are contact contributions.

Fig. 2 Diagrams (a) – (d), sometimes referred to as Born terms, give negative energy corrections, important in γ_5 theory. The processes (a) – (d) are included in the one body diagrams shown in Fig. $1a$–b, *if* these diagrams are evaluated exactly using a vertex function which is an exact solution of the bound state equation shown in (e). However, if negative energy contributions are neglected in the evaluation of the one body currents, then these contributions can be obtained approximately from the negative energy contributions to the virtual nucleon propagator (labeled N) in the Born diagrams.

Fig. 3 Form factors employed in the exchange-current operators as function of momentum transfer. The isovector combination $2F_1^V(Q^2) = F_1(Q^2, 1/2) - F_1(Q^2, -1/2)$ of nucleon form factors in the parametrization of Ref. 12 is shown in the solid curve, the isovector Sachs form factor G_E^V in the same parameterization is the dashed curve, the pion form factor is the dot-dashed curve, and the axial form factor of the nucleon is the dotted curve.

Fig. 4 Nonrelativistic calculations of the ^{3}He magnetic form factor, using different choices of elementary form factors for the pion exchange, pion contact term, and rho exchange terms. The short dashed line (with the largest secondary maximum) is the result of Ref. 13, which uses the common form factor $G_E^V(Q^2)$ for all exchange currents. The dotted curve shows the result when the realistic pion form factor, F_π, is substituted for G_E^V in the pion exchange term, the long dashed curve when F_C is substituted for G_E^V in the contact term, and the dot-dashed curve is the calculation when, in the respective terms, *both* F_π *and* F_C

are substituted for G_E^V. For comparison, the solid line shows the result when $F_1^V(Q^2)$ is used in place of $G_E^V(Q^2)$ for all three form factors. All calculations are based on the Paris potential with added single Δ-isobar excitation, i.e., on force model A2 of Ref. 13. All parameters in the exchange-current operators, i.e., the pion and rho cut-off masses Λ, are taken over unchanged from Ref. 13. The experimental data are taken from Ref. 14.

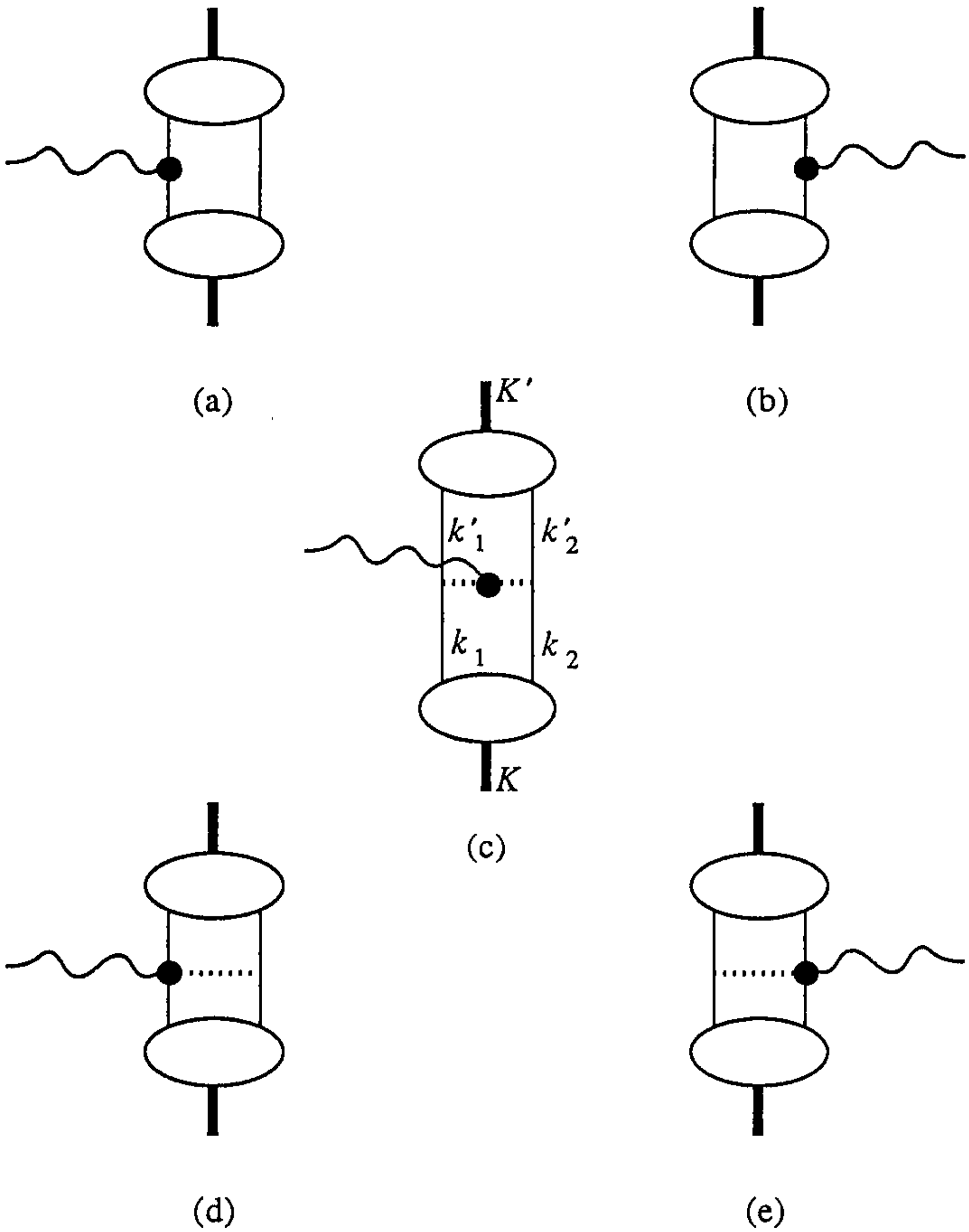

Fig. 1

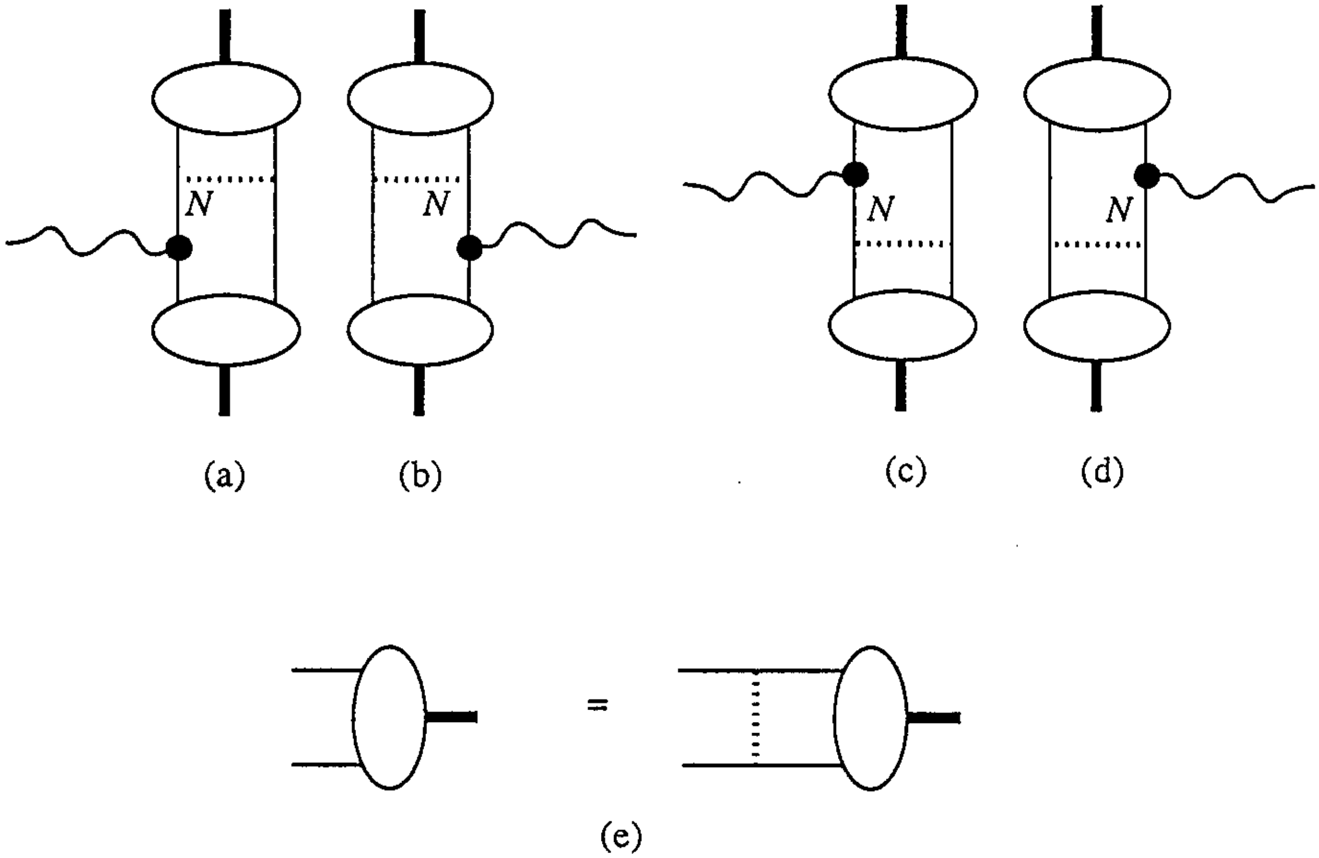

Fig. 2

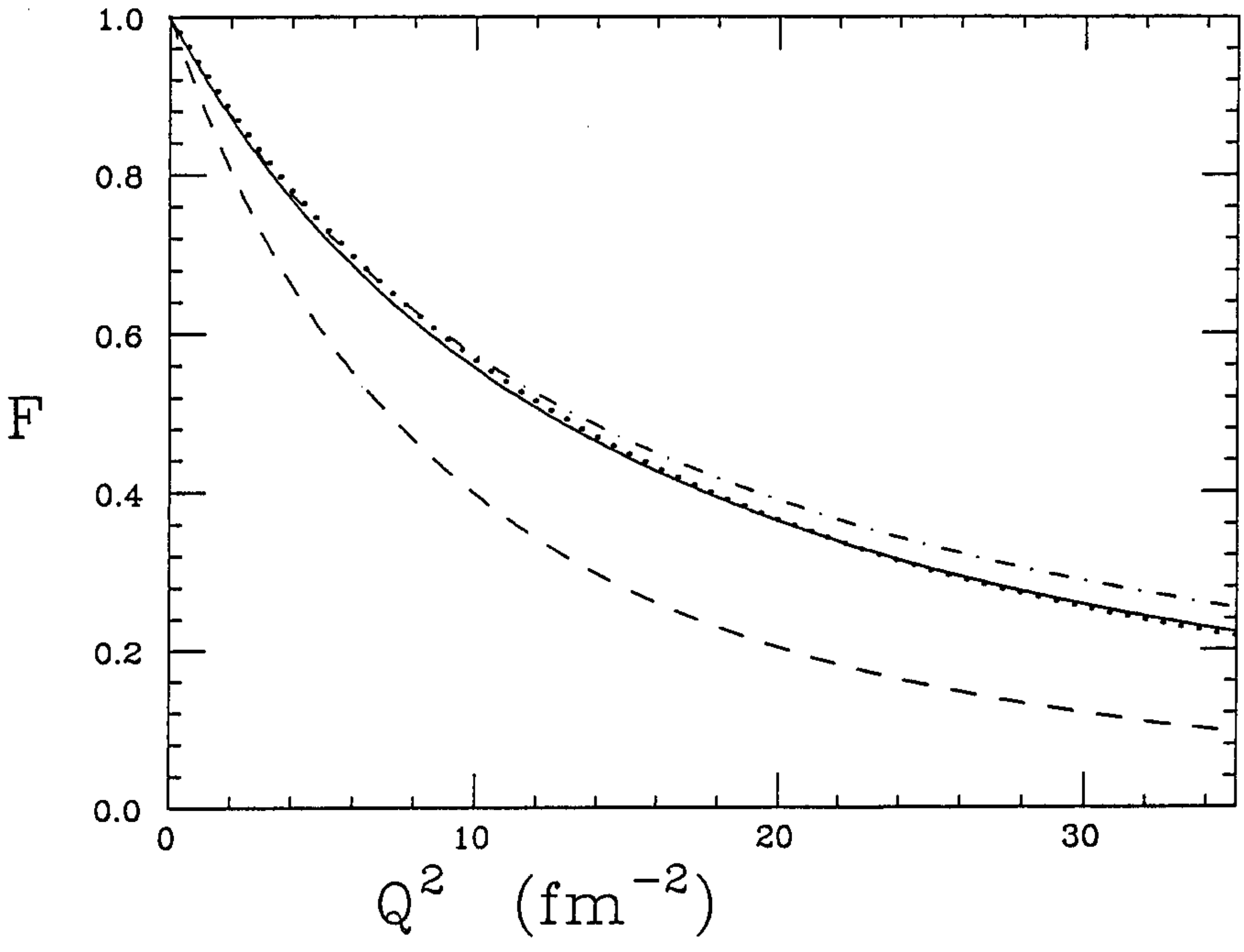

Fig. 3

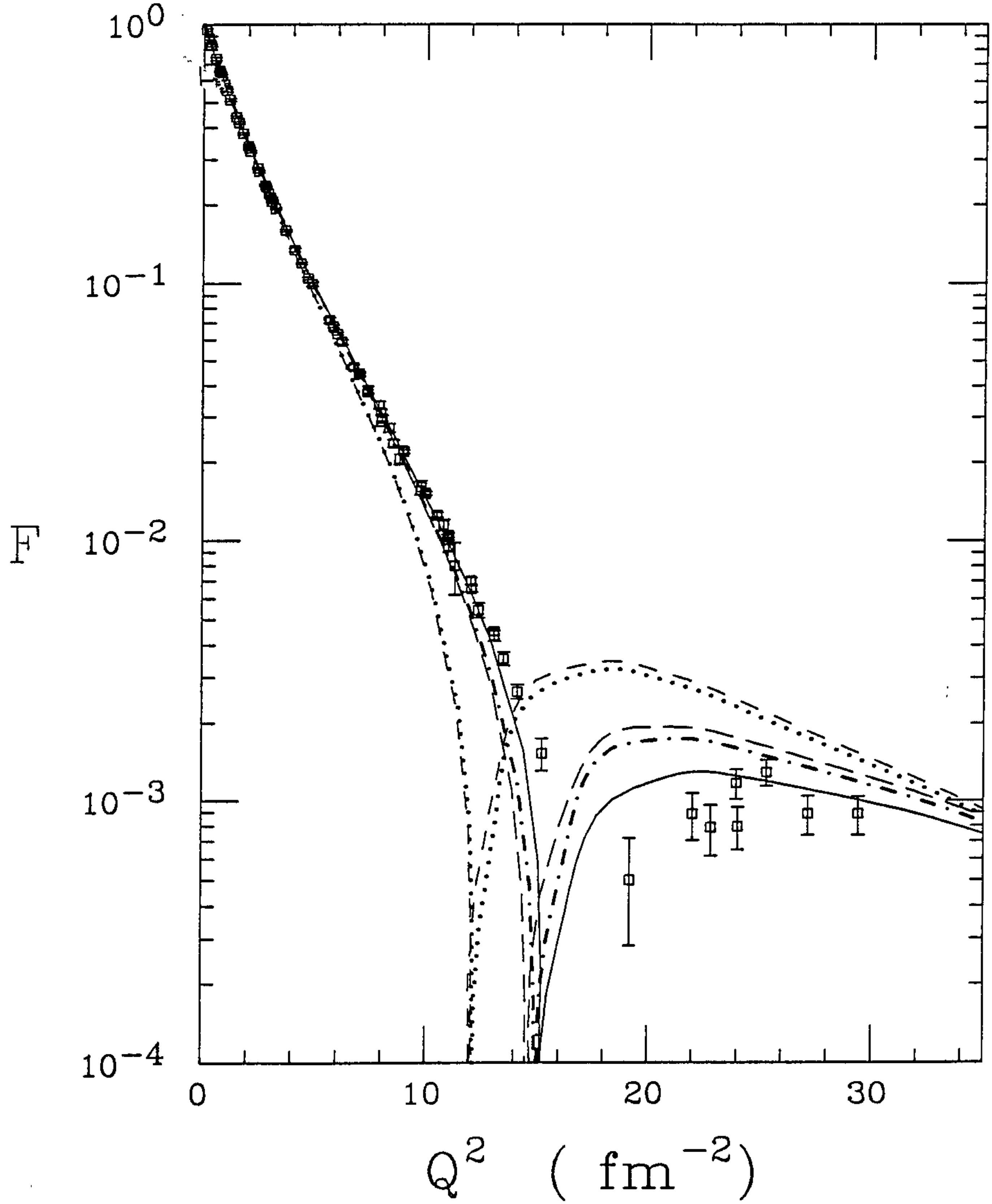

Fig 4